MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

PCL0016 – CALCULUS

(ALL Sections/Groups)

12 AUGUST 2022
9:00 A.M - 11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 5 pages including cover page with FOUR questions.

2. Attempt **ALL FOUR** questions and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 marks)

(a) Evaluate the following limits. (**Do not use** L'Hôpital's Rule):

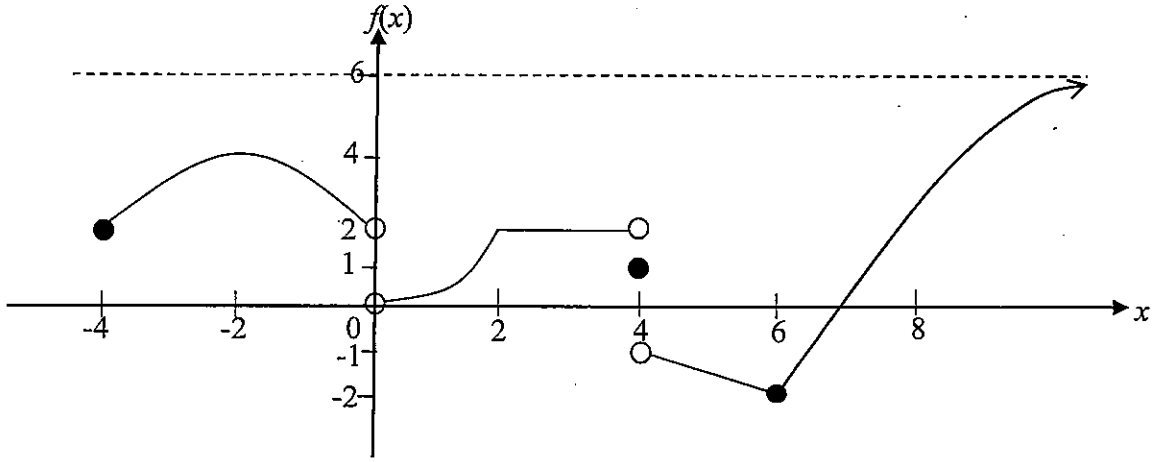
(i) $\lim_{x\to 2}\dfrac{x^2-4}{x^2-9x+14}$. [3 marks]

(ii) $\lim_{x\to\infty}\dfrac{3x^5+2x^3-4x+2}{2x^4-4x^3+6}$. [3 marks]

(iii) $\lim_{x\to 0}\dfrac{\sin 5x}{x}+\tan 2x$. [3 marks]

(iv) $\lim_{x\to 1}\dfrac{\sqrt{x^2+2}-\sqrt{3}}{x-1}$ [5 marks]

(b) The graph of a function $f(x)$ is given below.



Find the following limits:

(i) $\lim_{x\to 4^+} f(x)$ [1 mark]

(ii) $\lim_{x\to 0} f(x)$ [1 mark]

(iii) $\lim_{x\to 4^-} f(x)$ [1 mark]

(iv) $\lim_{x\to\infty} f(x)$ [1 mark]

(v) $\sqrt{\lim_{x\to -2} f(x)}+\lim_{x\to 6} f(x)$ [2 marks]

(c) Is the function $f(x)$ in part (b) continuous at $x=0$ and at $x=4$? Justify your answers by using the conditions for continuity at a point. [5 marks]

Continued.....

Question 2 (25 marks)

(a) Evaluate $\frac{dy}{dx}$ for the following functions:

(i) $y = x^5 - \frac{1}{x^3} + \pi e^x + 100$. [2 marks]

(ii) $y = x^2 \ln x + \tan(\sin x)$. [3 marks]

(iii) $y = \frac{x^2\sqrt{x+2}}{(x+3)(2x+1)^4}$ by using logarithmic differentiation. [7 marks]

(b) Evaluate the following integrals:

(i) $\int \frac{(x-3)\left(x^2 - x^{\frac{1}{2}}\right)}{x^{\frac{2}{3}}} dx$ [4 marks]

(ii) $\int \frac{\cos\left(\frac{1}{x^2}\right)}{4x^3} dx$ by using integration by substitution. [3 marks]

(iii) $\int \frac{x+2}{(x-1)(x+3)} dx$ by using partial fraction decomposition. [6 marks]

Continued.....

Question 3 (25 marks)

(a) A shaded region is bounded by $y^2 = x^4$, $x = 2$ and the x-axis as shown in **Figure 1**.

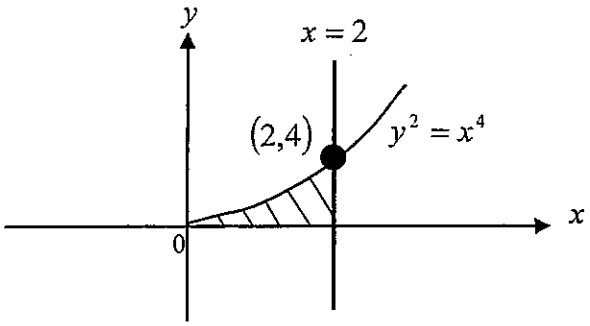


Figure 1

(i) Find the area of the shaded region. [3 marks]

(ii) By using the disk method, find the volume of the solid generated by revolving the shaded region about the x - axis. [4 marks]

(iii) By using the washer method, find the volume of the solid generated by revolving the shaded region about $y = 4$. [4 marks]

(iv) By using the cylindrical shell method, find the volume of the solid generated by revolving the shaded region about $y = -1$. [4 marks]

(b) Find the arc length of the curve $y = \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 1$ from $x = 1$ to $x = 2$. [7 marks]

(c) A spring has a natural length of 1m. A force 25N stretches the spring to a length of 1.5m. How much work will it take to stretch the spring 3m beyond its natural length? [3 marks]

Continued.....

Question 4 (25 marks)

(a) Solve the following differential equations.

(i) $\frac{dy}{dx} = \frac{\cos x}{e^y}$. [5 marks]

(ii) $\frac{dy}{dx} + y\tan x = \sec x$. [6 marks]

(b) Find the particular solutions of the following differential equations subject to the given initial values.

(i) $\left(\frac{y^2}{xe^x}\right)\frac{dy}{dx} = y, \quad y(0) = 2$. [7 marks]

(ii) $4y'' + 12y' + 9y = 0, \quad y(0) = 2, y'(0) = -1$ [7 marks]

End of Paper